Difficulties associated with the interpretation of K-G eq^n :

Since, $E = \pm\sqrt{p^2c^2 + m^2c^4}$ i.e.,
the total energy of the particle is +ve as well as negative.

This means that the probability $P(r,t)$ is not always +ve. Hence, it is no

Same as conventional positional probability density ($\psi^*\psi$) i.e., ψ is not the amplitude probability as obtained by Schroedinger eqⁿ.

This means that the wave funcⁿ ψ is not same as used in Schroedinger eqⁿ. This ψ is known as Field. not the wave funcⁿ.

According to new improv. interpretation of ψ by Pauli-Weisskopf (1934), P multiplied by e i.e. (eP) is interpretated as charge density

eP = charge density or electronic charge density,

e may be +ve or -ve, therefore eP can be positive and negative

eS = current density or electronic current density.

In K-G eqⁿ ψ has one component (scalar) i.e, ψ is real. If ψ has more components (more degree of freedom) then those components will show spin

Spin → degrees of freedom (up, down, no spin) 11.

K-G → 1 d.f. → scalar (spinless)
(only orbital exists)

motion. In the absence of other components K-G eqⁿ will describe particles of zero spin like π-mesons.

Positive and negative sign shows that the relativistic theories give rise to both particles and anti-particles.

If the ~~en~~ energy 'E' < mc^2, then the probability P in K-G eqⁿ reduces to non-relativistic probability density $\psi^*\psi$ as follows :-

density.

The wave solⁿ is given by:-

$$\Psi(r,t) = \psi(r)\, e^{-iEt/\hbar}$$

If E' is non-relativistic energy, then total energy E may be expressed as $E = E' + mc^2$; mc^2 being rest energy

$$\Psi(r,t) = \psi(r)\, e^{-i(E'+mc^2)t/\hbar}$$

$$= \psi(r)\, e^{-E't/\hbar} \cdot e^{-imc^2t/\hbar}$$

$$\psi(r,t) = \psi'(r,t)\, e^{-imc^2t/\hbar} \quad (19)$$

differentiate (19) w.r.t 't'

$$\frac{\partial \psi}{\partial t} = \left(\frac{\partial \psi'}{\partial t} - \frac{imc^2}{\hbar}\psi'\right) e^{-imc^2t/\hbar} \quad (20)$$

Taking complex conjugate of (19) and (20), we get

$$\psi^*(r,t) = \psi'^*(r,t)\, e^{icm^2t/\hbar}$$

$$\frac{\partial \psi^*}{\partial t} = \left(\frac{\partial \psi'^*}{\partial t} + \frac{imc^2}{\hbar}\psi'^*\right) e^{imc^2t/\hbar} \quad (21)$$

The probability is given by:-

$$P(r,t) = \frac{\hbar}{2imc^2}\left(\psi \frac{\partial \psi^*}{\partial t} - \psi^* \frac{\partial \psi}{\partial t}\right)$$

Substituting these values, we get

$$P(r,t) = \frac{\hbar}{2imc^2}\left\{\psi'(r,t)\, e^{-imc^2t/\hbar} \left\{\left(\frac{\partial \psi'^*}{\partial t}\right.\right.\right.$$

$\frac{imc^2}{\hbar}\psi'^*\Big)e^{imc^2t/\hbar}\Big\} - \{\psi$ by $\psi^+_{h_{\prime\prime}}$ Hermitian.

$\Big\{\Big(\frac{\partial\psi'}{\partial t} - \frac{imc^2}{\hbar}\psi'\Big)e^{-imc^2t/\hbar}\Big\}\Big]$

$$= \frac{\hbar}{2imc^2}\Big[\psi'(r,t)\Big(\frac{\partial\psi'}{\partial t} + \frac{imc^2}{\hbar}\psi'^*\Big) - \psi'^*(r,t)\Big(\frac{\partial\psi'}{\partial t} - \frac{imc^2}{\hbar}\psi'\Big)\Big]$$

$$= \frac{1}{2mc^2}\Big[\psi'(r,t)\Big(\frac{\hbar}{i}\frac{\partial\psi'^*}{\partial t}\Big) - \psi'^*\Big(i\hbar\frac{\partial\psi'}{\partial t}\Big)\Big] + \psi'^*\psi'$$

$$= \frac{1}{2mc^2}\Big[\psi'(r,t)\Big(-i\hbar\frac{\partial\psi'^*}{\partial t}\Big) + \psi'^*\Big(i\hbar\frac{\partial\psi'}{\partial t}\Big)\Big] + \psi'^*\psi'$$

$$= \frac{E'}{mc^2}\psi'^*\psi' + \psi'^*\psi'$$

(Since $E'^* = E'$, eigen value of H being real)

$$= \psi'^*\psi'$$

(Since non relativistic energy $E' << mc^2$)

which is correct non-relativistic expression for probability density.